

$$\max U(c, h) = \ln(c) + \alpha \ln(1-h)$$

$$\text{s.t. } c \leq y \xrightarrow{\text{equality}} c = y = zh$$

$$L = \ln(c) + \alpha \ln(1-h) + \lambda(zh - c)$$

$$\text{FOC}_c \rightarrow \frac{1}{c} - \lambda = 0 \rightarrow \lambda = \frac{1}{c}$$

$$\text{FOC}_h \rightarrow \frac{-\alpha}{1-h} + \lambda z = 0 \rightarrow \lambda = \frac{\alpha}{(1-h)z}$$

$$\frac{1}{c} = \frac{\alpha}{(1-h)z} \rightarrow c = \frac{z(1-h)}{\alpha}$$

$$\text{FOC}_\lambda \rightarrow c = zh \rightarrow \frac{z(1-h)}{\alpha} = zh \rightarrow 1-h = \alpha h \rightarrow h = \frac{1}{1+\alpha}$$

$$c = zh \rightarrow c = \frac{z}{1+\alpha}$$

$$\max U(c, h) = \ln(c) + \alpha \ln(1-h)$$

$$\text{s.t. } c + G \leq y \quad (y = zh)$$

$$L = \ln(c) + \alpha \ln(1-h) + \lambda(zh - G - c)$$

$$\text{FOC}_c \rightarrow \lambda = \frac{1}{c}$$

$$\text{FOC}_h \rightarrow \lambda = \frac{\alpha}{(1-h)z}$$

$$c = \frac{z(1-h)}{\alpha}$$

$$\text{FOC}_\lambda \rightarrow c + G = zh \rightarrow \frac{z(1-h)}{\alpha} + G = zh \rightarrow z\left(\frac{\alpha+1}{\alpha}\right)h = G + \frac{z}{\alpha} \rightarrow h = \frac{\alpha G + z}{z(\alpha+1)}$$

$$c + G = zh \rightarrow c + G = z\left(\frac{\alpha G + z}{z(\alpha+1)}\right) \rightarrow c = \frac{\alpha G + z}{\alpha+1} - G \rightarrow c = \frac{z-G}{\alpha+1}$$

$$\max U(c, L) = c - \beta L$$

$$\text{s.t. } c \leq L^\alpha$$

$$L = c - \beta L + \lambda (L^\alpha - c)$$

$$\frac{\partial L}{\partial c} = 1 - \lambda \leq 0, \quad c \frac{\partial L}{\partial c} = 0$$

$$\frac{\partial L}{\partial L} = -\beta + \alpha \lambda L^{\alpha-1} \leq 0, \quad L \frac{\partial L}{\partial L} = 0$$

$$\frac{\partial L}{\partial \lambda} = L^\alpha - c \geq 0, \quad \lambda \frac{\partial L}{\partial \lambda} = 0$$

$$c, L > 0$$

(الف) باید برای زنده ماندن غذا مصرف کند و برای تولید غذا باید کار کند پس داریم:

$$\begin{aligned} \frac{\partial L}{\partial c} = 0 &\rightarrow \lambda = 1 \\ \frac{\partial L}{\partial L} = 0 &\rightarrow \lambda = \frac{\beta}{\alpha} L^{1-\alpha} \end{aligned} \rightarrow \frac{\beta}{\alpha} L^{1-\alpha} = 1 \rightarrow L = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}}$$

$$\lambda = 1 \rightarrow \frac{\partial L}{\partial \lambda} = 0 \rightarrow c = L^\alpha \rightarrow c = \left(\frac{\alpha}{\beta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$L = c = 0$$

(ب) در این حالت داریم:

$$\frac{\partial L}{\partial c} \leq 0 \rightarrow 1 - \lambda \leq 0 \rightarrow \lambda \geq 1 \rightarrow \frac{\partial L}{\partial \lambda} = 0 \rightarrow L^\alpha - c \geq 0 \xrightarrow{L=c=0} 0 \geq 0 \checkmark$$

$$\frac{\partial L}{\partial L} \leq 0 \rightarrow -\beta + \alpha \lambda L^{\alpha-1} \leq 0 \xrightarrow{L=0} \beta \geq 0 \checkmark$$

بله این یک تعادل است.

$$\max c - \beta L$$

$$\text{s.t. } c \leq L^\alpha$$

$$c \geq \left(\frac{\gamma \beta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}$$

$$L = c - \beta L + \lambda (L^\alpha - c) + \mu \left(c - \left(\frac{\gamma \beta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right)$$

$$\frac{\partial L}{\partial c} = 1 - \lambda + \mu = 0 \rightarrow \lambda = \mu + 1$$

$$\frac{\partial L}{\partial L} = -\beta + \alpha \lambda L^{\alpha-1} = 0$$

$$\frac{\partial L}{\partial \lambda} = L^\alpha - c \geq 0, \quad \lambda \frac{\partial L}{\partial \lambda} = 0$$

$$\frac{\partial L}{\partial \mu} = c - \left(\frac{\gamma \beta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \geq 0, \quad \mu \frac{\partial L}{\partial \mu} = 0$$

$$\text{if } \lambda = 0 \rightarrow \mu = -1, \quad \beta = 0 \rightarrow c - \left(\frac{\gamma \beta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} = 0 \rightarrow c = 0 \quad \times$$

$$\text{if } \mu = 0 \rightarrow \lambda = 1, \quad L = \left(\frac{\beta}{\alpha}\right)^{\frac{1}{1-\alpha}} \rightarrow L^\alpha - c = 0 \rightarrow c = \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\hookrightarrow c - \left(\frac{\gamma \beta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \geq 0 \rightarrow \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \geq \left(\frac{\gamma \beta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \rightarrow \alpha > 1 \quad \leftarrow$$

$$\text{if } \mu \neq 0, \quad \lambda \neq 0 \rightarrow L^\alpha = c = \left(\frac{\gamma \beta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \rightarrow L = \left(\frac{\gamma \beta}{\alpha}\right)^{\frac{1}{1-\alpha}}$$

$$\beta = \alpha \lambda \left(\frac{\alpha}{\gamma \beta}\right) \rightarrow \lambda = \frac{\gamma \beta^\gamma}{\alpha^\gamma} \rightarrow \mu = \frac{\gamma \beta^\gamma}{\alpha^\gamma} - 1 \rightarrow \alpha \neq \sqrt[\gamma]{\gamma} \beta \quad \leftarrow$$